ELECTRO-ACOUSTICAL NOISE

R.J.J. Zijlstra and W. Westera

Fysisch Laboratorium, Rijksuniversiteit Utrecht, The Netherlands

1. INTRODUCTION

Electro-acoustical noise is the noise associated with the interaction of free carriers with acoustic waves in piezoelectric semiconductors. Increased interest in these physical phenomena was triggered by the experimental results on acoustic amplification in CdS reported in 1961 by Hutson, McFee and White [1]. The electric field accompanying an acoustic wave, be it a bulk or a surface wave, in a piezoelectric semiconductor produces periodic variation in the electric potential. Free electrons, interacting with these waves, tend to seek the potential minima and in doing so get dragged along by the propagating wave. If the drift velocity of the carriers is smaller than the sound velocity the acoustically trapped carriers receive a net accelerating force from the wave. Acoustic energy is thereby transferred from the wave to free electrons and the wave is attenuated. However, if an applied electric field causes the drift velocity of electrons to become larger than the sound velocity, then energy is transferred from electrons to the wave while it is travelling in the direction of the drifting carriers. As a result the acoustic waves are amplified. White [2] in 1962 gave a linear description of this electro-acoustic effect, known as the linear small signal gain theory. This effect also applies to spontaneously generated waves with thermal amplitudes. These amplified acoustic waves also interact with the free carriers. This gives rise to essentially non-linear effects, which may include parametric interaction of acoustic waves, current saturation, large current noise and domain formation [3,4]. These effects cannot be described by White's theory. Several authors [5,6] have tried to describe these phenomena. Since, however, it is very difficult to describe these non-linear effects when starting from basic principles, many of these phenomena are not yet understood quantitatively. Moore realized that the observed current saturation and current fluctuations are caused by the trapping of bunches of free charge carriers in potential troughs that are associated with acoustic waves amplified from the thermal background. Accordingly, he ascribed the observed current noise in CdS to fluctuations in the creation-annihilation processes of these potential troughs [7]. The expression for the noise spectrum thus obtained gave a reasonable explanation for Moore's experimental data. Subsequently Friedman [8] and Nakamura [9,10] using different approaches were less successful in describing these experimental data.

In our opinion Moore's model is essentially correct and scored highest in the description of experimental data. Zijlstra and Gielen [11] extended Moore's theory by accounting for transit time effects in a local description. However, they neglected the displacement and the diffusion current in the expression for the current density and assumed that the creation and annihilation of troughs are independent of electric field strength. Their calculation resulted among other things in a frequency independent impedance. Experimentally, however, the observed ac impedance of electro-acoustically active CdS crystals turned out to be frequency dependent [12]. The observed frequency dependence of the ac impedance could be explained, however, by taking into account diffusion, space charge, displacement current and the electric field dependence of the trough creation and annihilation rates [13,14].

In section 2 we present theoretical results along the lines indicated. Section 3 is concerned with experimental results on electro-acoustical noise in CdS.

2. THEORY

Essentially we shall use Moore's model but extend it as indicated in the introduction. In addition we shall use the so-called phenomenological, hydrodynamic theory of electron absorption and emission of sound in semiconductors. Usually the criterion for the validity of this approach is written as $ql \ll 1$ and $\omega\tau \ll 1$ [14], where q is wave number of acoustic waves, l is free carrier mean free path, τ is their collision time and ω is angular frequency. Gulyaev and Kozorezov [15] argue, however, that the mean free path l should be replaced by $\sqrt{D\tau_\zeta}$ and the collision time by τ_ζ, where τ_ζ is the energy relaxation time and D is the free carrier diffusion constant.

2.1. DESCRIPTION OF THE MODEL AND BASIC EQUATIONS

We consider an n-type homogeneous piezoelectric semiconducting crystal, where the electric field is applied along a symmetry axis, the x-axis. The sample is provided with two ohmic contacts with spacing L. It can be shown for this case that the formulas can be put in a one-dimensional form [14]. Therefore we shall start straightaway with a one-dimensional description.

In the analysis we use the following sign convention, $E < 0$ and $I < 0$; then $V > 0$, where E is the electric field strength, I is the electric current and V is de applied voltage.

Let n_d be the local density of the free electrons in the conduction band and n_s the density of electrons trapped in potential troughs that are associated with acoustic waves amplified from the thermal background. The total density of electrons in the conduction band is then given by $n = n_d + n_s$. Gauss's equation yields

$$\partial D/\partial x = -\, q(n - \bar{n}) \quad , \qquad (1)$$

where D is the dielectric displacement, $-q$ the electron charge and where steady-state values are indicated by bar; $\bar{n}$ is assumed to be equal to the thermal equilibrium density of free carriers. The total current density then is

$$j = -\, q n_d v_d - q n_s v_s' + q D_n(\partial n/\partial x) + \partial D/\partial t \quad , \qquad (2)$$

where D_n is the diffusion constant of electrons, v_d is the drift velocity of free electrons and v_s' is the x-component of the sound velocity v_s. Since j is solenoidal, we have $\partial j/\partial x = 0$. The piezoelectric relations read

$$T = cS - eE \quad ; \qquad D = \varepsilon E + eS \quad . \qquad (3)$$

According to Newton's second law

$$\partial T/\partial x = \rho(\partial^2 u/\partial t^2) \qquad (4)$$

with $S = \partial u/\partial x$, where T and S are the stress and strain, whereas c, e and ε in eq (3) are the elastic, piezoelectric and dielectric constants respectively, ρ is the mass density and u the displacement. It should be borne in mind that the constants in the piezoelectric relation are related to elastic, piezoelectric and dielectric tensor elements valid for an anisotropic solid [14].

When p and b are the creation and annihilation rates of troughs per unit volume respectively, the master equation for the trough density n_t reads

$$\partial n_t/\partial t = p - b - v_s'\,(\partial n_t/\partial x) \quad . \qquad (5)$$

In addition we assume that each trough contains N electrons, independent of position x and electric field strength E. Then the density of trapped electrons becomes $n_s = N\, n_t$.

2.2. THE STATIONARY STATE

From eq (1), (3) and (4) it follows for the steady state that $\partial\bar{D}/\partial x = (1 + K_e^2)(\partial\bar{E}/\partial x) = 0$ where $K_e = (e^2/\varepsilon c)^{\frac{1}{2}}$ is the electromechanical coupling constant, which is usually much smaller than one. Thence $\bar{E} = -\,\bar{V}/L$ and $\bar{v}_d = \bar{V}/L$ is independent of position. It follows from eq (3) and (4) that $v_s'(\partial\bar{n}_s/\partial x) = \bar{v}_d(\partial\bar{n}_s/\partial x)$ and since $\bar{v}_d \neq v_s'$ in the electroacoustically active regime we have $(\partial\bar{n}_s/\partial x) = (\partial\bar{n}_d/\partial x) = 0$. Finally for the current we obtain $\bar{I} = -\, qA\bar{n}_s v_s' - qA\bar{n}_d(\mu\bar{V}/L)$, where A is the contact area. Note that the mobility μ as well as $\bar{n}_s$ and $\bar{n}_d$ may depend on electric field strength at high applied fields.

2.3. AC BEHAVIOUR AND NOISE

In order to describe the ac behaviour and fluctuations around a steady state we linearize the equations from section 2.1. Then the linearized current equation becomes

$$\Delta j = \sigma \,\Delta E + q(\bar{v}_d - \bar{v}_s{}')\Delta n_s + qD_n \frac{\partial \Delta n}{\partial x} - q\bar{v}_d \Delta n + \frac{\partial \Delta D}{\partial t} \qquad (2a)$$

where $\sigma = q\mu\bar{n}_d$. The linearized rate equations for the troughs extended by a Langevin source function H, which formally describes the fluctuations in the creation and annihilation, reads:

$$\frac{\partial \Delta n_t}{\partial t} + \frac{\Delta n_t}{\tau} = \beta \Delta E - \bar{v}_s{}' \frac{\partial \Delta n_t}{\partial x} + H \qquad (5a)$$

where $\tau^{-1} = -[\partial(p-b)/\partial n_t]_{\Delta n_t = 0}$, $\beta = [\partial(p-b)/\partial E]_{\Delta E = 0}$ and $H \equiv \Delta(p-b)_{\Delta E = \Delta n_t = \Delta n_s = 0}$.

Note that we finally have 11 linearized equations with 14 variables. We are interested in a spectral decomposition of the fluctuations. We Fourier transform the equations with respect to time. Then 11 equations result with 13 variables. These 11 equations allow one in principle to eliminate 10 variables so that 3 remain. If we do not consider fluctuations (H = 0) 2 variables remain, say $\tilde{j}$ and $\tilde{E}$ (Fourier transforms are denoted by tilde). Thence $\tilde{j}$ can be expressed in $\tilde{E}$ and in the ac voltage $\tilde{V} = -\int_0^L \tilde{E}dx$. As a result the ac impedance can be calculated.

If one considers an ac open circuit by putting $\tilde{j} = 0$, then $\tilde{V}$ can be expressed in $\tilde{H}$ and the voltage fluctuations can be calculated in terms of Langevin source terms.

2.4. THE AC IMPEDANCE

Carrying out the programme for the calculation of the ac impedance as indicated in section 2.1 and using the boundary conditions $\tilde{T}(0) = \tilde{T}(L) = 0$, corresponding to free end surfaces, one finds [13,14]

$$Z(\omega) = \frac{L}{A\left(\frac{\alpha\tau}{1+i\omega\tau} + \sigma + i\omega\varepsilon\right)} \{1 + K_e{}^2 f(\omega)\} \quad , \qquad (6)$$

where $\alpha = q(\bar{v}_d - \bar{v}_s{}')N\beta$ and $f(\omega)$ is a complicated function of frequency [13,14]. Substituting reasonable values for the unknowns in eq (6) we found that the second term between brackets only contributes significantly at frequencies given by

$$f = (2m+1)\bar{v}_s{}'/2L \qquad m = 0, 1, 2, \ldots\ldots \qquad (7)$$

Four limiting cases are of interest

(i) If we put $\alpha = 0$ and $\sigma = q\mu\bar{n}$, in other words we neglect the presence of troughs, we have the linear small signal approximation, and eq (6) reduces to an expression derived by Greebe [17].

(ii) If $\omega \to 0$, then eq (6) reduces to the differential resistance: $Z(0) = L/A(\alpha\tau + \sigma)$.

(iii) If $\omega \gg |\sigma + \alpha\tau/(1+i\omega\tau)|/\varepsilon$, then $Z(\omega) \approx L/iA\omega\varepsilon$, corresponding to the geometrical capacitance of the device.

(iv) If $\sigma \gg |i\omega\varepsilon + \alpha\tau/(1+i\omega\tau)|$, then we find $Z(\omega) \sim L/A\sigma$.

Note that $\alpha < 0$ if the number of troughs increases with increasing applied voltage.
It follows that the low frequency plateau value of the impedance is a factor $\sigma/(\sigma+\alpha\tau)$ larger than the intermediate frequency plateau value.

In addition to the ac impedance the following expression could be found for the imaginary part of the wavenumber of waves travelling in the direction of drifting particles:

$$\alpha_e = \frac{1}{2} K_e^2 \frac{\omega_c}{v_s} \frac{\gamma^*}{\gamma^{*2} + (\frac{\omega}{\omega_D} + \frac{\omega_c}{\omega})^2} \tag{8}$$

where v_s = sound velocity, $\omega_D = v_s^2/D_n$ and $\omega_c = (\alpha\tau + \sigma)/\varepsilon$; $\gamma^* = 1 - (\vec{v}_d \cdot \vec{v}_s)/v_s^2$. These expressions reduce to White's result for the linear attenuation coefficient if the differential conductivity is replaced by the ohmic one. The frequency of maximum amplification, ω_m, is given by

$$\omega_m^2 = \omega_c \omega_D \quad . \tag{9}$$

For a discussion of the influence of traps and magnetic fields on the electro-acoustic effect we refer to the literature [3,18].

2.5. NOISE

The noise problem has been solved only by ignoring diffusion and displacement current, by assuming space charge neutrality and by assuming that the number of electrons, N, trapped per trough is independent of position and electric field strength. By Fourier transforming eq (5a) with respect to time and using $\tilde{n}_s = N\,\tilde{n}_t$ one then finds

$$\frac{d\tilde{n}_s}{dx} + \frac{1}{\tau v_s'} (1 + i\omega\tau)\tilde{n}_s = \frac{\beta N}{v_s'} \tilde{E} + \frac{N}{v_s'} \tilde{H} \quad . \tag{5b}$$

The Fourier transformed eq (2a) reads

/$\bar{v}_d$

$$\tilde{j} = \sigma\tilde{E} + q(\bar{n}_d - v_s')\tilde{n}_s \quad . \tag{2b}$$

If one considers an ac open circuit by putting $\tilde{j} = 0$, $\tilde{n}_s$ can be eliminated from these equations and the following differential equation in $\tilde{E}$ results:

$$\frac{d\tilde{E}}{dx} + \Lambda\tilde{E} = -\frac{qN\eta}{\sigma} \tilde{H} \quad , \tag{10}$$

where $\Lambda = (\tau_1^{-1} + i\omega)/v_s'$, $\tau_1^{-1} = \tau^{-1} + \alpha/\sigma$ and $\eta = (\bar{v}_d - v_s')/v_s'$. If we assume that the end surfaces are kept free, then $\tilde{T}(0) = \tilde{T}(L) = 0$, which for this case also implies $\tilde{E}(0) = \tilde{E}(L)$. Using these boundary conditions and remembering that $S_V(f)$ is proportional to $<\tilde{V}\cdot\tilde{V}^*>$, where $\tilde{V} = -\int_0^L \tilde{E}\, dx$, one finds by integrating eq (10)

$$S_V(f) = \frac{1}{\Lambda\Lambda^*} [\frac{qN\eta}{\sigma}]^2 \int_0^L\int_0^L S_H(x_1,x_2,f)dx_1dx_2 \quad . \tag{11}$$

If it is assumed that creation and annihilation processes occur spontaneously and that a δ-function space correlation exists, we have $S_H(x_1,x_2,f) = (4\bar{b}/A)\delta(x_1 - x_2)$, where we used $\bar{p} = \bar{b}$. Substituting this into eq (11) yields:

$$S_V(f) = \frac{4\bar{b}Lv_s'^2\tau_1^2}{A(1 + \omega^2\tau_1^2)} \frac{q^2N^2\eta^2}{\sigma^2} \quad . \tag{12}$$

The outcome of the calculations is a Lorentzian noise spectrum. It should be remembered, however, that the outcome depends on the boundary conditions and on the rather stringent restriction on the space correlation of the fluctuations in the creation and annihilation processes. The latter condition is obviously included for mathematical convenience. It seems reasonable to assume that physically these conditions are met if $\omega \ll \omega_m$. If the ac impedance Z is calculated using the same conditions as for the noise calculations one finds

$$|Z| = \frac{L}{A\sigma} \; \frac{\tau_1}{\tau} \left(\frac{1 + \omega^2\tau^2}{1 + \omega^2\tau_1^2}\right)^{\frac{1}{2}} \quad . \tag{13}$$

If we introduce the trough transit time $\tau_t = L/v_s'$ and assume that $\overline{b} = \overline{n}_t/\tau$, then the saturation current $I_s = AqN\overline{b}\tau v_s'$ and then it follows from eqs (12) and (13) that

$$S_I(f) = 4NqI_s \, \frac{\eta^2\tau/\tau_t}{1 + \omega^2\tau^2} \quad . \tag{14}$$

3. EXPERIMENTAL RESULTS

Some experimental results obtained recently with CdS [16] crystal bars will be presented. The small faces, typically of the order of 1 mm^2, were provided with indium evaporated ohmic contacts; the lengths of the bars varied from 1.4-2.8 mm. The c-axis of the hexagonal crystals was in the length direction. In this shape the crystal bars could be used for Brillouin scattering as well as for electrical measurements. The crystals were semiconducting with resistivities of the order of 1 Ωm. The length of the bars was kept smaller than 3 mm in order to suppress travelling electro-acoustic domain formation and the accompanying oscillatory behaviour. To avoid excessive Joule heating of the samples the high voltage was applied in voltage pulses of 40 μs duration with a repetition rate of 4 Hz,

Figure 1 shows a typical current-voltage (I-V) characteristic. At low applied voltage the sample is ohmic, whereas at higher voltages non-ohmic conduction occurs because electrons are trapped in potential troughs associated with acoustic waves moving at the sound velocity. In figure 2 the spectral intensity of the ac short-circuited current noise, S_I, at 10 MHz is plotted double logarithmically versus applied voltage. One distinguishes a rise with slope 2 at low voltages due to generation recombination noise, followed by a very sharp rise because of the occurrence of the electro-acoustic effect and finally a less sharp rise which is even followed by a decrease with increasing voltage.

The threshold voltage V_c for the occurrence of electro-acoustic current fluctuations can be determined [12] by an extrapolation of the steep curve down to the thermal current noise level. The thus obtained values for V_c invariably turned out to be somewhat lower than the knee voltage of the I-V characteristic due to the fact that in the transition region between ohmic and non-ohmic linear behaviour the density of trapped electrons is field dependent. In the past there has been a controversy about whether longitudinal or transverse waves are responsible for the onset of the electro-acoustic effect. Particularly electrical data alone left some room for speculation. This was partly due to the wide spread in the electron mobility values reported in the literature and to the occurrence of trapping effects, which were not taken into account [7,12].

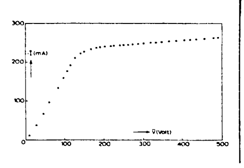

Fig. 1 Current versus applied voltage for a CdS crystal with the electric field applied along the c-axis.

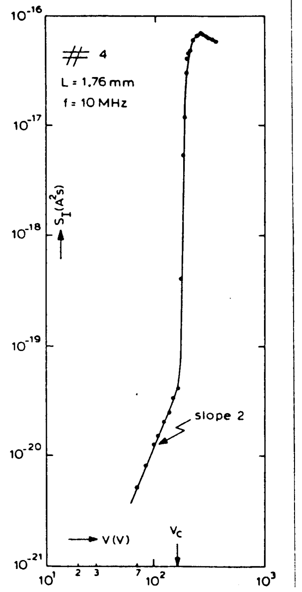

Fig. 2 The spectral current noise intensity S_I at 10 MHz versus applied voltage. The critical voltage V_c for the onset of the electro-acoustic effect is indicated by an arrow.

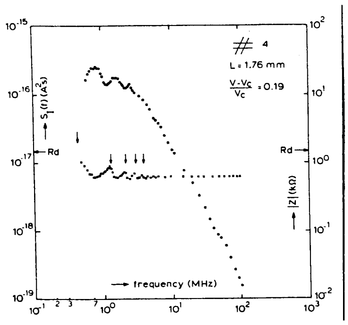

Fig. 3 The current noise intensity $S_I(f)$ and the absolute value of the ac impedance Z vs frequency. Resonances in $|Z|$ are indicated by arrows.

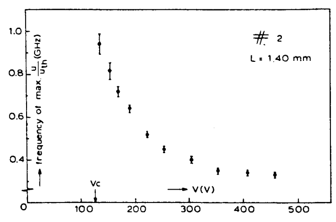

Fig. 4 The frequency where the relative acoustic energy density is at its maximum is plotted versus applied voltage. The voltage V_c marking the onset of the electro-acoustic effect is indicated by an arrow.

By combining results of electrical measurements on relatively low ohmic samples with good ohmic contacts with Brillouin scattering data on the same samples Westera [14,16] settled the dispute by proving that transverse waves were responsible for the onset of the electro-acoustic effect. The measured threshold voltage (cf. figure 2) corresponded to a sound velocity of transversal on-axis waves of 1.77×10^3 ms^{-1} and a room temperature mobility of 2.20×10^{-2} $m^2V^{-1}s^{-1}$, in agreement with a value for the mobility reported by Spear and Mort [19]. Figure 3 shows S_I and the absolute value of the ac impedance $|Z|$ versus frequency for the same sample at $(V - V_c)/V_c = 0.19$. The spectrum has a Lorentzian frequency dependence with some modulations and can apart from these modulations be interpreted with eq (14). The observed cross-over frequency corresponds to an average life time for troughs of 5.3×10^{-8} s. From the low frequency plateau and a saturation current of 160 mA (deduced from the IV-characteristic) one then finds with the help of eq (14) that N is about 10^6. $|Z|$ is almost frequency independent for frequencies larger than 10 MHz, whereas with decreasing frequency it gradually rises towards the value of the differential resistance. In addition resonances are observed at the odd harmonics of 450 Hz. The impedance data can be well interpreted with eqs (6) and (13) with $\tau_1/\tau = 2.4$ and provided that the velocity occurring in eq (7) can be interpreted as the x-component of a group velocity that corresponds to a phase velocity with an off-axis angle of 25° of a transverse wave, which is in good agreement with Keller [20] and San'ya et al. [21]. The very sharp peaks predicted by eq (6) are apparently smoothed out because of the occurrence of a distribution of waves with different off-axis angles. It should be noted that minima occur in the noise spectrum at the resonance frequencies of the impedance. However, spectral noise intensities often differ markedly from a Lorentzian. This is supposedly due to the fact that the δ-function space correlation for the trough creation and annihilation fluctuations no longer holds. Finally in figure 4 the frequency f_m, where the ratio of electro-acoustic to thermal acoustic energy has a maximum, as determined from Brillouin scattering data, is plotted versus applied voltage. It was found that the behaviour of f_m is in good agreement with eq (9).

REFERENCES

[1] A.R. Hutson, J.M. McFee and D.L. White, *Phys. Rev. Lett.* 7, 237 (1961).
[2] D.L. White, *J. Appl. Phys.* 33, 2547 (1962).
[3] N.I. Meyer and M.H. Jørgensen, *Festkörperprobleme X*, Ed. O. Madelung, Pergamon Vieweg (1970) p. 21.
[4] H. Kuzmany, *Phys. Stat. Sol.* 25a, 9 (1974).
[5] G. Johri and H.N. Spector, *Phys. Rev.* B14, 4955 (1977).
[6] P.K. Tien, *Phys. Rev.* 171, 970 (1968).
[7] A.R. Moore, *J. Appl. Phys.* 38, 2327 (1967).
[8] L. Friedman, *Phys. Rev.* 174, 737 (1968).
[9] K. Nakamura, *J. Phys. Soc. Japan* 39, 860 (1975).
[10] K. Nakamura, *J. Phys. Soc. Japan* 40, 350 (1976).
[11] R.J.J. Zijlstra and P.A. Gielen, *Physica* 95B, 190 (1978).
[12] P.A. Gielen and R.J.J. Zijlstra, *Physica* 95B, 347 (1978).

[13] W. Westera, R.J.J. Zijlstra and M.A. van Dijk, *Phys. Lett.* 78A, 371 (1980).
[14] W. Westera and R.J.J. Zijlstra, *Physica* 106B, to be published June 1981.
[15] Y.V. Gulguaev and A.G. Kozorezov, *Sol. State Commun.* 27, 375 (1978).
[16] W. Westera and R.J.J. Zijlstra, *Physica*, to be published.
[17] C.A.A.J. Greebe, *Philips Res. Repts.* 20, 1 (1965).
[18] R.M. White, *High-Frequency Ultrasonic Devices, Topics in Solid State and Quantum Electronics*, Ed. W.D. Hershberger, Wiley, London, (1972).
[19] W.W. Spear and J. Mort, *Proc. Phys. Soc.* 81, 130 (1963).
[20] D. Keller, *Phys. Stat. Sol.* 16a, 87 (1973).
[21] M. San'ya, M. Yamada, C. Hamaguchi and J. Nakai, *Jap. J. Appl. Phys.* 13, 611 (1974).