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THEORY OF ELECTRO-ACOUSTIC EFFECTS IN HEXAGONAL PIEZOELECTRIC SEMICONDUCTORS

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A theoretical model is presented to describe electro-acoustic effects in single crystalline materials. The calculations, which are essentially linear, are based on the assumption that the electro-acoustic effects can be described by the trapping of bunches of free charge carriers in two types of potential troughs. One type of trough is associated with acoustic waves, which are amplified from the thermal background, travelling in the direction of the drifting carriers; the second type is associated with waves with large amplitudes travelling in the opposite direction. It is assumed that the two types of troughs are independent, and that they are created and annihilated at random throughout the crystal.

Expressions are derived for the *IV*-characteristic, the current noise, the ac impedance and the wave attenuation coefficients. In these calculations the anisotropy of the crystal is taken into account. When space charge, diffusion and the displacement current are neglected, the calculated noise spectra consist of two Lorentzians. The ac impedance, which is calculated without making these approximations, shows two low-frequency roll-offs and resonances that are related to the transit time of potential troughs. Finally some remarks on dispersion effects are made.

1. Introduction

The first theoretical description of the electro-acoustic effect, i.e. the amplification of travelling acoustic waves in piezoelectric semiconductors where an electric drift field is applied, was given in 1962 by White [1] in the form of a linear classical continuum theory. Some years later Brillouin scattering studies of the growth of the acoustic flux in semiconducting CdS [2, 3] showed that this linear theory applies only in the weak acoustic flux region. At higher acoustic flux intensities, however, the frequency of maximum amplification of acoustic waves was found to be an order of magnitude lower than predicted by the theory. The failure of the linear theory is due to the appearance of essentially non-linear effects at higher acoustic flux intensities. These effects may include current saturation, large current fluctuations, ac-impedance effects, parametric down-conversion of waves, electro-acoustic domain formation, current oscillations and so on [4, 5]. We shall direct our attention mainly to current saturation, current fluctuations and ac-impedance effects.

Several authors [6, 7] have tried to describe the non-linear effects. Since, however, this is very difficult when starting from basic principles, many of these phenomena are not yet understood. Already in 1962 Smith [8] proposed that the observed current saturation is due to the bunching of free charge carriers in potential troughs, which are coupled to the amplified acoustic waves viá the piezoelectric effect. Accordingly, in 1967 Moore [9] avoided the need for non-linear equations to describe the large current fluctuations by suggesting that these fluctuations are caused by the random creation and annihilation of potential troughs. The expression obtained by Moore for the current noise spectrum gave a reasonable explanation for his experimental data. Subsequently Friedman [10] and Nakamura [11, 12], using different approaches, were less successful in describing these experimental data.

In 1978 Zijlstra and Gielen [13] modified

Moore's calculation by accounting for transit time effects in a one-dimensional local description. They neglected the dielectric displacement and the diffusion current in the expression for the current density. In addition they assumed space-charge neutrality and also assumed that the creation and annihilation rates of troughs are independent of the electric field strength. The resulting transit time effects in the current noise, however, are very sensitive to the choice of the boundary conditions. Since the boundary conditions which they chose imply an inconsistency at low frequencies, their calculated result for the current noise should be considered doubtful. Furthermore, their calculation resulted in a frequency-independent impedance. Experimentally, however, the ac impedance of electro-acoustically active CdS turned out to be frequency dependent [14].

In 1965 Greebe [15] used the linear small-signal gain theory to calculate the effects that boundary conditions have on the impedance of semiconducting piezoelectric plates. His results, however, were not applicable to electro-acoustically active semiconductors.

Recently Westera et al. [5, 16] derived an expression for the ac impedance, starting from the trough model, while taking into account diffusion, space charge, the displacement current, the electric field dependence of the trough creation and annihilation rates and anisotropy effects. The result was in good agreement with the experimental data on CdS. It was found, however, that the relaxation times derived from additional current noise spectra [17] differed markedly from those obtained from the ac impedance [18]. This inconsistency can be removed by introducing into the theory two types of potential troughs, with two corresponding relaxation times. This theory, which will be treated here, is again a local description and essentially linear. Furthermore, anisotropy and dispersion effects are taken into account. It should be noted that this theory holds for crystals where a continuous amplified acoustic flux is present (i.e. where no travelling electro-acoustic domains occur). For a comparison of this theory with experimental results on CdS, the reader is referred to refs. [17–19].

In section 2 the basic equations for our calculations are introduced. In section 3 the equations for the stationary state are given. In section 4 the spectral current noise intensity is calculated. An expression for the ac impedance is derived in section 5. In section 6 expressions for the acoustic attenuation coefficients are given; some concluding remarks on the effects of dispersion of the acoustic waves are presented in section 7.

2. Basic equations

A classical continuum description is applicable, if it is assumed that the free carrier intercollision time is small on the time-scale considered, and the mean free path of free charge carriers is much smaller than the wavelengths involved in the sound amplification process.

We consider an n-type homogeneous piezoelectric semiconducting crystal, where the electric field is applied along a symmetry axis, the x_3 axis. In the case of CdS the x_3 axis coincides with the c axis. Together with the x_1 and x_2 axes the x_3 axis forms a Cartesian coordinate system. The sample is provided with ohmic contacts at $x_3 = 0$ and $x_3 = L$, where L is the contact spacing.

When the drift velocity of the electrons exceeds the sound velocity, acoustic waves, originating from the thermal background and travelling in the direction of the drifting electrons, are amplified. As a result potential troughs which propagate with the sound velocity are spontaneously created and annihilated throughout the crystal. Note that the creation and annihilation occurs at random because of the incoherence of the thermal background waves.

Since it is known from the literature [16, 20] that under these conditions transverse off-axis waves are amplified rather than longitudinal on-axis waves, our calculation must include the anisotropic propagation characteristics of acoustic waves as well as the anisotropy of the piezoelectric and dielectric properties. We shall consider acoustic waves with an arbitrary polarization and propagation direction and show how the general form of the piezoelectric relations and the wave equation can be reduced to the more simple one-dimensional form (cf. ref. [4]).

There is experimental evidence [17, 18] that two relaxation mechanisms are involved in the electric phenomena to be described. Relaxation times obtained from current noise spectra are considerably smaller than those obtained from ac-impedance measurements. Therefore we shall introduce two types of potential troughs, each with a different decay time.

If the electric field is applied along the c axis, the amplification (or attenuation) coefficient for acoustic waves is a function of the angle between the wave vector and the c axis, the off-axis angle δ [4, 21]. Therefore, acoustic waves with wave vectors lying on a cone centred around the c axis, with a half-cone angle δ, are amplified to the same extent. If the crystal end-surface at $x_3 = L$ (anode) acts as a perfect mirror, the wave vectors of reflected acoustic waves also lie on a cone, with the same half-cone angle δ. Although the amplification coefficient of acoustic waves is generally not a delta function of the off-axis angle, it is in practice sufficiently peaked at a favourable piezoelectrically active direction [19, 21] to allow us to consider waves travelling in this direction only. Therefore, the stationary acoustic energy distribution is built up by forward travelling (amplified) waves and backward travelling (attenuated) waves, with the same preferential off-axis angle δ.

It should be noticed that the ultimate acoustic amplitudes near the cathode may largely exceed the amplitudes of the original thermal waves, because of net acoustic round-trip gain. McFee [22] observed net acoustic round-trip gain during the build-up of the acoustic flux in CdS. His observations were in accordance with White's linear theory [1]. When the net round-trip gain is reduced to unity due to some non-linear loss mechanism a stationary state is reached.

We now introduce two types of potential troughs: one type of trough is formed by forward travelling acoustic waves, the second by backward travelling waves. In ref. [18] it is shown that the trough velocity is given by the group velocity of the amplified (and attenuated) acoustic waves. As a result of the elastic anisotropy and the electro-acoustic dispersion (cf. section 7, ref. [20]) the direction and magnitude of the group velocity may differ markedly from the direction and magnitude of the phase velocity.

To simplify the calculation we consider a system of acoustic waves, which travel in a fixed wave vector direction $\boldsymbol{\kappa}$ only. Ultimately we are only interested in the motion of the troughs projected on the x_3 axis.

From White's calculation [1] it appears that piezoelectric stiffening has only a slight effect on the phase velocity of acoustic waves. Therefore, acoustic waves travelling in opposite directions have approximately the same absolute phase velocity. This conclusion also applies to the group velocity. If the velocity of the potential troughs associated with forward travelling acoustic waves is denoted by $\boldsymbol{v}_g$, the backward travelling troughs in our system will thus travel with $-\boldsymbol{v}_g$. For the theoretical description the nature of v_g is irrelevant; it can be either a phase or a group velocity.

In the following analysis we use the sign convention: $V > 0$ while $E < 0$ and $I < 0$, where V is the applied voltage, E the electric field strength and I the electric current.

Let n_d be the local, instantaneous density of free electrons in the conduction band, and n_{s_1} and n_{s_2} the local, instantaneous density of electrons trapped in potential troughs which travel towards the anode and the cathode, respectively. When the total local electron density in the conduction band is denoted by n, we have

$$n = n_d + n_{s_1} + n_{s_2} . \tag{1}$$

Gauss's equation yields

$$\frac{\partial D_i}{\partial x_i} = -q(n - \bar{n}), \tag{2}$$

where D_i is the ith component of the dielectric displacement vector, x_i the corresponding component of the position vector and $-q$ the electron charge. The time average of n is denoted by $\bar{n}$ and is assumed to be equal to the thermal equilibrium density of free charge carriers. This assumption, which implies that there is no space charge in the stationary state, is supported by potential probe measurements, which showed that the electric field is uniform even in the electro-acoustically active regime [8, 9]. It should be noted that in eq. (2) we have to carry out a summation over repeated subscripts (Einstein convention); here i is a subscript running from 1 to 3. The equation for the total current density j for our system becomes

$$j_i = -qn_d v_{d_i} - qn_{s_1} v_{g_i} + qn_{s_2} v_{g_i} + qD_{n_{ij}} \frac{\partial n_d}{\partial x_j} + \frac{\partial D_i}{\partial t}, \tag{3}$$

where $D_{n_{ij}}$ are tensor elements describing the anisotropic diffusion, t is time, $\boldsymbol{v}_d$ the drift velocity and $\pm\boldsymbol{v}_g$ the velocity of the potential troughs. Here we have assumed that only the free carriers can contribute to diffusion and that the diffusion tensor elements are equal to their thermal equilibrium values.

In addition, since j is solenoidal, we have,

$$\frac{\partial j_i}{\partial x_i} = 0, \tag{4}$$

the piezoelectric relations

$$T_{ij} = c_{ijkl} S_{kl} - e_{kij} E_k, \tag{5}$$

$$D_i = \varepsilon_{ij} E_j + e_{ijk} S_{jk}, \tag{6}$$

Newton's second law:

$$\frac{\partial T_{ik}}{\partial x_k} = \rho \frac{\partial^2 u_i}{\partial t^2}, \tag{7}$$

and

$$S_{ij} = \tfrac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \tag{8}$$

where T_{ij} and S_{ij} are elements of the stress and strain tensor, respectively, and c_{ijkl}, e_{ijk} and ε_{ij} are the elastic, piezoelectric and dielectric tensor elements, respectively. $\boldsymbol{E}$ is the electric field strength vector, ρ the mass density and $\boldsymbol{u}$ the spatial displacement vector.

If p_1 and b_1 are the creation and annihilation rates per unit volume of forward travelling troughs, respectively, the master equation for the density of these troughs, n_{t_1}, reads

$$\frac{\partial n_{t_1}}{\partial t} = p_1 - b_1 - v_{g_i} \frac{\partial n_{t_1}}{\partial x_i}. \tag{9}$$

Analogously, the master equation for the density of backward travelling troughs, n_{t_2}, reads

$$\frac{\partial n_{t_2}}{\partial t} = p_2 - b_2 + v_{g_i} \frac{\partial n_{t_2}}{\partial x_i}. \tag{10}$$

In addition we make the approximation that each forward travelling trough contains N_1 electrons and each backward travelling trough N_2 electrons, where N_1 and N_2 do not depend explicitly on $\boldsymbol{x}$. Then the densities of trapped electrons become

$$n_{s_1} = N_1 n_{t_1} \tag{11}$$

and

$$n_{s_2} = N_2 n_{t_2}. \tag{12}$$

3. The stationary state

If the electric field strength is applied along the x_3 axis we have

$$\frac{\partial \bar{E}_k}{\partial x_j} = 0 \qquad \text{unless } k = j = 3\,. \tag{13}$$

Because of the crystal symmetry it is reasonable to assume that the static spatial displacement and strain components, $\bar{u}_i$ and $\bar{S}_{ij}$, depend only on x_3. Then we obtain (cf. eq. (8))

$$\bar{S}_{ij} = 0 \qquad \text{unless } i = 3 \text{ or } j = 3\,, \tag{14}$$

and

$$\frac{\partial \bar{S}_{ij}}{\partial x_k} = 0 \qquad \text{unless } k = 3\,. \tag{15}$$

In the stationary state eq. (7) yields

$$\frac{\partial \bar{T}_{ij}}{\partial x_j} = 0\,. \tag{16}$$

With the help of eqs. (5) and (16) we find

$$c_{ijkl}\frac{\partial \bar{S}_{kl}}{\partial x_j} - e_{kij}\frac{\partial \bar{E}_k}{\partial x_j} = 0\,. \tag{17}$$

Furthermore, in crystals with a hexagonal symmetry, such as those of CdS, we have $c_{333k} = c_{33k3} = 0$, unless $k = 3$. With the help of eqs. (13)–(15), for $i = 3$ eq. (17) then becomes

$$\frac{\partial \bar{E}_3}{\partial x_3} = \frac{c_{3333}}{e_{333}}\frac{\partial \bar{S}_{33}}{\partial x_3}\,. \tag{18}$$

From eqs. (2) and (6) we find

$$\varepsilon_{ij}\frac{\partial \bar{E}_j}{\partial x_i} + e_{ijk}\frac{\partial}{\partial x_i}\bar{S}_{jk} = 0\,. \tag{19}$$

Since for hexagonal crystals $e_{33k} = e_{3k3} = 0$ unless $k = 3$, from eq. (19) with the help of eqs. (13)–(15) we obtain

$$\frac{\partial \bar{E}_3}{\partial x_3} = \frac{e_{333}}{\varepsilon_{33}}\frac{\partial}{\partial x_3}\bar{S}_{33}\,. \tag{20}$$

From eqs. (18) and (20) we can conclude that

$$\frac{\partial \bar{E}_3}{\partial x_3} = \frac{\partial}{\partial x_3}\bar{S}_{33} = 0\,. \tag{21}$$

Note that this result is a consequence of the assumption of space-charge neutrality in the stationary state (cf. eq. (2)).

Eq. (21), among other things, means that we are dealing with a uniform electric field strength $\bar{E}_3$. Thence $\bar{v}_{d_3}$ is also independent of x_3, so

$$\bar{v}_{d_3} = -\mu_{33}\bar{E}_3 = \mu_{33}\frac{\bar{V}}{L}\,, \tag{22}$$

where μ_{ij} are the free electron mobility tensor elements and $\bar{V}$ is the voltage applied to the sample.

Since the sample is assumed to be homogeneous, we can put

$$\frac{\partial \bar{n}}{\partial x_i} = 0\,. \tag{23}$$

In the stationary state the diffusion contribution to the current density can be omitted. In view of the translational symmetry along the x_1 and x_2 axes we can write

$$\frac{\partial \bar{n}_d}{\partial x_i} = \frac{\partial \bar{n}_{s_1}}{\partial x_i} = \frac{\partial \bar{n}_{s_2}}{\partial x_i} = 0 \qquad \text{for } i = 1, 2\,. \tag{24}$$

Combining eqs. (1)–(4), (23) and (24) yields

$$(\bar{v}_{d_3} - v_{g3})\frac{\partial \bar{n}_{s_1}}{\partial x_3} = -(\bar{v}_{d_3} + v_{g3})\frac{\partial \bar{n}_{s_2}}{\partial x_3}\,. \tag{25}$$

If we assume that the kinetics of the two types of troughs are statistically independent, it follows that the only possible solution of eq. (25) is given by

$$\frac{\partial \bar{n}_{s_1}}{\partial x_3} = \frac{\partial \bar{n}_{s_2}}{\partial x_3} = \frac{\partial \bar{n}_d}{\partial x_3} \equiv 0 . \tag{26}$$

The assumption of the statistical independence of the two types of troughs is inspired by the idea that the creation and annihilation of potential troughs is solely determined by the random distribution of acoustic wave amplitudes and phases. Note that for the derivation of eq. (26) the assumption of space-charge neutrality in the stationary state (cf. eq. (2)) again plays an important role.

If we use eqs. (3), (4), (22), (23) and (26) we find for the electric current in the stationary state the following expression:

$$\bar{I} = -(qA\mu_{33}\bar{n}_d/L)\bar{V} - qAv_{g3}(\bar{n}_{s_1} - \bar{n}_{s_2}) , \tag{27}$$

where A is the cross-sectional area of the crystal (= the contact area). Expression (27) is also valid when the field strength is below the threshold for amplification, since in that case $\bar{n}_d = \bar{n}$ and $\bar{n}_{s_1} = \bar{n}_{s_2} = 0$. It then follows that eq. (27) reduces to Ohm's law. (It should be noted that $\bar{n}_d$, $\bar{n}_{s_1}$, $\bar{n}_{s_2}$ and v_{g3} generally depend on $\bar{V}$.)

For amplification of an acoustic wave with off-axis angle δ, the component of the drift velocity $\boldsymbol{v}_d$ along the phase velocity $\boldsymbol{v}_s(\delta)$ should exceed the phase velocity [4, 21]. With eq. (22) this condition for electro-acoustic amplification is given by

$$\bar{V} > v_s(\delta)L/\cos(\delta)\, \mu_{33} , \tag{28}$$

where $v_s(\delta) = |\boldsymbol{v}_s(\delta)|$. In the case of CdS [20, 21], or ZnO [20], the threshold voltage V_c for the amplification of sound waves is determined by on-axis waves ($\delta = 0$), according to

$$V_c = v_s(0)L/\mu_{33} . \tag{29}$$

Eq. (29) holds for both longitudinal and transverse piezoelectrically active sound waves, although the electromechanical coupling factor for the transverse on-axis waves happens to be zero. For transverse waves, the off-axis angle of maximum amplification increases at increasing voltage until it saturates at 30° (in CdS and ZnO). This occurs because there is a maximum in the electromechanical coupling factor at this angle [4, 21]. This implies that v_{g3} occurring in eq. (27) depends essentially on the applied voltage $\bar{V}$. As will be shown in section 7, electro-acoustic dispersion may cause v_{g3}, if $\boldsymbol{v}_g$ is identified with the group velocity, to become voltage dependent even at a fixed off-axis angle.

Finally we find from the master equations (eqs. (9) and (10)), and from eqs. (11), (12) and (24), that the stationary state values of the creation and annihilation rates are equal:

$$\bar{p}_1 = \bar{b}_1 \tag{30}$$

and

$$\bar{p}_2 = \bar{b}_2 . \tag{31}$$

4. The current noise

In this section we present the calculation of the spectral current noise intensity. To simplify the calculation we assume that space-charge neutrality prevails. This assumption seems valid on a time-scale which is large compared to the dielectric relaxation time. Furthermore, diffusion and the displacement current are neglected. Due to these assumptions the spectral current noise intensity can be calculated without using the piezoelectric relations (eqs. (5) and (6)), Newton's second law (eq. (7)) and eq. (8). This implies that the elastic, dielectric and piezoelectric anisotropy is not taken into account explicitly. For a further discussion of the validity of these assumptions we refer to the end of section 5.

We recall that the basic equations presented in section 2 hold for a system of waves travelling in a fixed wave vector direction $\boldsymbol{\kappa}$. In general $\boldsymbol{\kappa}$ is characterized by an off-axis angle δ and an azimuthal angle ϕ. Because we neglect space charge, diffusion and the displacement current, the calculation can be simplified substantially by averaging the fluctuating quantities a priori over the azimuthal angle ϕ. Thus the contribution of all wave vectors lying on a cone, with half-cone angle δ, centred around the c axis is taken into account. Furthermore, the fluctuating quantities are averaged over the cross-sectional area. The averaged quantities obtained in this way are indicated by $'$. So when $y(x_1, x_2, x_3, \phi, \delta, t)$ is a fluctuating local quantity, $y'(x_3, \delta, t)$ is defined by

$$y' = \frac{1}{A} \int\int_A \left\{ \frac{1}{2\pi} \int_0^{2\pi} y \, d\phi \right\} dx_1 \, dx_2 \,. \tag{32}$$

From this consideration it follows that y' is independent of x_1, x_2 and ϕ. In the following we shall rewrite our equations in terms of these averaged quantities. Note that these quantities are still local with respect to x_3.

For small deviations Δ from the stationary state we find that by linearizing eq. (3) and using eq. (32), for $i = 3$,

$$\Delta j_3' = -q\bar{n}_d \Delta v_{d_3}' - q\bar{v}_{d_3} \Delta n_d' - q v_{g_3} \Delta n_{s_1}' + q v_{g_3} \Delta n_{s_2}' \,. \tag{33}$$

Here we have neglected the ac-conductivity contribution of trapped electrons. With the help of eq. (1) space-charge neutrality yields

$$\Delta n_d' = -\Delta n_{s_1}' - \Delta n_{s_2}' \,. \tag{34}$$

Because of the symmetry around the x_3 axis, we can write

$$\Delta v_{d_3}' = -\mu_{33} \Delta E_3' \,. \tag{35}$$

From eqs. (33), (34) and (35) we obtain

$$\Delta j_3' = q(\bar{v}_{d_3} - v_{g_3}) \Delta n_{s_1}' + q(\bar{v}_{d_3} + v_{g_3}) \Delta n_{s_2}' + q\mu_{33} \bar{n}_d \Delta E_3' \,. \tag{36}$$

For the fluctuations in the creation and annihilation rates of forward travelling troughs, we can write, in first-order approximation,

$$\Delta p_1' - \Delta b_1' = -\frac{\Delta n_{t_1}'}{\tau_1} + \left[\frac{\partial}{\partial E_3} (p_1 - b_1) \right]_{\Delta E_3 = 0} \Delta E_3' + L_{p_1}' - L_{b_1}' \,, \tag{37}$$

where

$$\tau_1^{-1} = -\left[\frac{\partial}{\partial n_{t_1}} (p_1 - b_1) \right]_{\Delta n_{t_1} = 0} ,$$

τ_1 is the mean lifetime of the fluctuations in the density of forward travelling troughs, and

$$L_{p_1}' = [\Delta p_1']_{\substack{\Delta n_{t_2} = 0 \\ \Delta E_3 = 0}} \quad \text{and} \quad L_{b_1}' = [\Delta b_1']_{\substack{\Delta n_{t_1} = 0 \\ \Delta E_3 = 0}}$$

are Langevin source functions, which formally describe the spontaneous, random fluctuations in the creation and annihilation rates, respectively. Analogously, for backward travelling troughs we can write

$$\Delta p_2' - \Delta b_2' = -\frac{\Delta n_{t_2}}{\tau_2} + \left[\frac{\partial}{\partial E_3} (p_2 - b_2) \right]_{\Delta E_3 = 0} \Delta E_3' + L_{p_2}' - L_{b_2}' \,, \tag{38}$$

where

$$\tau_2^{-1} = -\left[\frac{\partial}{\partial n_{t_2}} (p_2 - b_2) \right]_{\Delta n_{t_2} = 0}$$

and

$$L_{p_2}' = [\Delta p_2']_{\substack{\Delta n_{t_2} = 0 \\ \Delta E_3 = 0}} \quad \text{and} \quad L_{b_2}' = [\Delta b_2']_{\substack{\Delta n_{t_2} = 0 \\ \Delta E_3 = 0}}$$

In eqs. (37) and (38) we have accounted for the field dependence of the creation and annihilation rates.

Although the number of electrons per trough, N_1 and N_2, might be functions of the electric field strength $\boldsymbol{E}$, so that local fluctuations $\Delta\boldsymbol{E}$ would induce local fluctuations ΔN_1 and ΔN_2, we assume that the fluctuations ΔN_1 and ΔN_2 can be neglected. This assumption is reasonable since the sudden trapping and de-trapping of a whole group ($N_1, N_2 \gg 1$) of electrons is the main reason for the occurrence of current fluctuations. Thence eqs. (11) and (12) yield

$$\Delta n'_{s_1} = N_1 \Delta n'_{t_1} \tag{39}$$

and

$$\Delta n'_{s_2} = N_2 \Delta n'_{t_2} . \tag{40}$$

From eqs. (9), (37) and (39) we obtain

$$\frac{\partial}{\partial t}\Delta n'_{s_1} = -\frac{\Delta n'_{s_1}}{\tau_1} + N_1\left[\frac{\partial}{\partial E_3}(p_1 - b_1)\right]_{\Delta E_3=0}\Delta E'_3 - v_{g3}\frac{\partial}{\partial x_3}\Delta n'_{s_1} + N_1(L'_{p_1} - L'_{b_1}) . \tag{41}$$

Eqs. (10), (38) and (40) yield

$$\frac{\partial}{\partial t}\Delta n'_{s_2} = -\frac{\Delta n'_{s_2}}{\tau_2} + N_2\left[\frac{\partial}{\partial E_3}(p_2 - b_2)\right]_{\Delta E_3=0}\Delta E'_3 + v_{g3}\frac{\partial}{\partial x_3}\Delta n'_{s_2} + N_2(L'_{p_2} - L'_{b_2}) . \tag{42}$$

Making a Fourier analysis, from eqs. (36), (41) and (42) we obtain (Fourier transformed quantities are denoted by tilde)

$$\begin{aligned}\tilde{j}'_3 = {} & \frac{qN_1(\bar{v}_{d_3} - v_{g3})}{(1/\tau_1 + i\omega)}(\tilde{L}'_{p_1} - \tilde{L}'_{b_1}) + \frac{qN_2(\bar{v}_{d_3} + v_{g3})}{(1/\tau_2 + i\omega)}(\tilde{L}'_{p_2} - \tilde{L}'_{b_2}) \\ & - \frac{q(\bar{v}_{d_3} - v_{g3})}{(1/\tau_1 + i\omega)} v_{g3}\frac{d}{dx_3}\tilde{n}'_{s_1} + \frac{q(\bar{v}_{d_3} + v_{g3})}{(1/\tau_2 + i\omega)} v_{g3}\frac{d}{dx_3}\tilde{n}'_{s_2} \\ & + \left\{\frac{qN_1(\bar{v}_{d_3} - v_{g3})}{(1/\tau_1 + i\omega)}\left[\frac{\partial}{\partial E_3}(p_1 - b_1)\right]_{\Delta E_3=0} + \frac{qN_2(\bar{v}_{d_3} + v_{g3})}{(1/\tau_2 + i\omega)}\left[\frac{\partial}{\partial E_3}(p_2 - b_2)\right]_{\Delta E_3=0} + q\mu_{33}\bar{n}_d\right\}\tilde{E}'_3 .\end{aligned} \tag{43}$$

Before this first-order differential equation is solved, it is advisable to choose suitable boundary conditions. Mechanical boundary conditions, such as zero stress at the boundaries corresponding to free end-surfaces, must be translated into boundary conditions for the electron densities if they are to be useful in eq. (43). This can be done by using the space-charge-neutrality condition. From eq. (8) it follows that at all positions (cf. eq. (12))

$$\Delta S'_{ij} = 0 \qquad \text{unless } i = 3 \text{ or } j = 3 . \tag{44}$$

Considering free end-surfaces we put

$$\Delta T'_{ij}(x_3) = 0 \qquad \text{at } x_3 = 0 \text{ and } x_3 = L . \tag{45}$$

For $i = j = 3$ it then follows from eq. (5) that

$$c_{3333}\Delta S'_{33}(x_3) - e_{333}\Delta E'_3(x_3) = 0 \quad \text{at } x_3 = 0 \text{ and } x_3 = L . \tag{46}$$

Here we have used $c_{33k3} = c_{333k} = 0$ unless $k = 3$, and $e_{k33} = 0$ unless $k = 3$, in crystals with a hexagonal symmetry. From Gauss's equation (eq. (2)) we obtain in the case of space-charge neutrality

$$\frac{\partial}{\partial x_3}\Delta D'_3(x_3) = 0 \qquad \text{or} \qquad \Delta D'_3(0) = \Delta D'_3(L) . \tag{47}$$

With eq. (6) it then follows that

$$\varepsilon_{33}\Delta E'_3(0) + e_{333}\Delta S'_{33}(0) = \varepsilon_{33}\Delta E'_3(L) + e_{333}\Delta S'_{33}(L) . \tag{48}$$

Here, we have used $\varepsilon_{3k} = 0$ unless $k = 3$, and

$e_{33k} = e_{3k3} = 0$ unless $k = 3$, in hexagonal crystals.
Combining eqs. (46) and (48) we find

$$\Delta E_3'(0) = \Delta E_3'(L) \,. \tag{49}$$

This electrical boundary condition corresponds to the mechanical boundary condition in eq. (45) for $i = j = 3$.

From eq. (4) we have

$$\frac{\partial}{\partial x_3} \Delta j_3' = 0 \,. \tag{50}$$

By using eqs. (36), (49) and (50) we find

$$(\bar{v}_{d_3} - v_{g_3})(\Delta n_{s_1}'(L) - \Delta n_{s_1}'(0)) = -(\bar{v}_{d_3} + v_{g_3})(\Delta n_{s_2}'(L) - \Delta n_{s_2}'(0)) \,. \tag{51}$$

Since we assumed that the two types of potential troughs are statistically independent, it follows that fluctuations $\Delta n_{s_1}'$ cannot be affected by fluctuations $\Delta n_{s_2}'$. Therefore, the only solution of eq. (51) is given by

$$\Delta n_{s_1}'(L) = \Delta n_{s_1}'(0) \,, \tag{52}$$

and

$$\Delta n_{s_2}'(L) = \Delta n_{s_2}'(0) \,. \tag{53}$$

In order to derive the ac short-circuited current fluctuations, we put

$$\tilde{V} = -\int_0^L \tilde{E}_3' \, \mathrm{d}x_3 \equiv 0 \,. \tag{54}$$

By integrating eq. (43) and using eqs. (50) and (52)–(54) we obtain the following solution for the Fourier transformed current fluctuations:

$$\tilde{I}_3 = A\tilde{j}_3' = \frac{qN_1(\bar{v}_{d_3} - v_{g_3})}{(1/\tau_1 + \mathrm{i}\omega)L} \times \int_0^L \iint_A (\tilde{L}_{p_1}'' - \tilde{L}_{b_1}'') \, \mathrm{d}x_1 \, \mathrm{d}x_2 \, \mathrm{d}x_3 + \frac{qN_2(\bar{v}_{d_3} + v_{g_3})}{(1/\tau_2 + \mathrm{i}\omega)L} \int_0^L \iint_A (\tilde{L}_{p_2}'' - \tilde{L}_{b_2}'') \, \mathrm{d}x_1 \, \mathrm{d}x_2 \, \mathrm{d}x_3 \,, \tag{55}$$

where the averaging of the local source functions over the azimuthal angle ϕ is denoted by $''$.

If the following assumptions are made:

(i) the fluctuations in L_{p_1}'', L_{p_2}'', L_{b_1}'' and L_{b_2}'' are uncorrelated;

(ii) there is a delta-function space correlation for each Langevin source function (L_{p_1}'', L_{p_2}'', L_{b_1}'', L_{b_2}'');

(iii) the noise in the source functions is shot noise;

for the spectral cross-intensities of fluctuations in $(L_{p_1}'' - L_{b_1}'')$ [13] we obtain (using $f = \omega/2\pi$)

$$S_{(L_{p_1}''-L_{b_1}'')}(\boldsymbol{x}, \boldsymbol{x}^*, f) = 4\bar{b}_1 \delta(x_1 - x_1^*)\delta(x_2 - x_2^*)\delta(x_3 - x_3^*) \,. \tag{56}$$

We used $\overline{p_1''} = \overline{b_1''} = \bar{b}_1$. Analogously, we find

$$S_{(L_{p_2}''-L_{b_2}'')}(\boldsymbol{x}, \boldsymbol{x}^*, f) = 4\bar{b}_2 \delta(x_1 - x_1^*)\delta(x_2 - x_2^*)\delta(x_3 - x_3^*) \,. \tag{57}$$

From eqs. (55)–(57) we obtain for the spectral current noise intensity S_I:

$$S_I(f) = 4\bar{b}_1 N_1^2 q^2 (\bar{v}_{d_3} - v_{g_3})^2 \frac{A}{L} \frac{\tau_1^2}{1 + \omega^2\tau_1^2} + 4\bar{b}_2 N_2^2 q^2 (\bar{v}_{d_3} + v_{g_3})^2 \frac{A}{L} \frac{\tau_2^2}{1 + \omega^2\tau_2^2} \,. \tag{58}$$

When we use (cf. ref. [13])

$$\bar{b}_1 = \bar{n}_{t_1}/\tau_1 = \bar{n}_{s_1}/N_1\tau_1 \,, \tag{59}$$

$$\bar{b}_2 = \bar{n}_{s_2}/N_2\tau_2 \,, \tag{60}$$

and

$$\tau_t = L/v_{g_3}\,, \tag{61}$$

τ_t being the transit time for the potential troughs, the expression for the spectral current noise intensity becomes

$$S_I(f) = 4qN_1(q\bar{n}_{s_1}v_{g_3}A)\left(\frac{\bar{v}_{d_3} - v_{g_3}}{v_{g_3}}\right)^2 \frac{\tau_1/\tau_t}{1+\omega^2\tau_1^2}$$

$$+4qN_2(q\bar{n}_{s_2}v_{g_3}A)\left(\frac{\bar{v}_{d_3} + v_{g_3}}{v_{g_3}}\right)^2 \frac{\tau_2/\tau_t}{1+\omega^2\tau_2^2}\,. \tag{62}$$

From eq. (62) we see that the current noise spectra contain two Lorentzian spectra with different weighting factors and, generally, different roll-off frequencies. It is concluded that at voltages slightly above V_c, where $\bar{v}_{d_3} \approx v_{g_3}$, the noise is caused mainly by the second term, which describes backward travelling troughs. At increasing voltage, both Lorentzians may contribute, depending on the relative magnitude of the weighting factors. As a consequence of considering an ac-shorted circuit, the field dependence of p_1, b_1, p_2 and b_2 has no influence on the current noise (cf. eqs. (43) and (54)). An important difference between these results and earlier calculations [9, 13] is, that $\bar{n}_{s_1}$ and $\bar{n}_{s_2}$ are allowed to be voltage dependent. It should be noted that, as indicated in section 3, v_{g_3} is voltage dependent as well. Because of these additional voltage dependences the magnitude of the current noise is generally not proportional to $(\bar{V} - V_c)^2$; earlier calculations [9, 13] predicted that the current noise is proportional to $(\bar{V} - V_c)^2$. Note that eq. (62) does not describe any potential trough transit time effects.

The result of the spectral current noise intensity, presented in eq. (62), suggests that the calculation can be simplified by considering total numbers of free and trapped electrons only (cf. ref. [9]). However, we believe that the treatment given in this section in terms of averaged local quantities is preferable, because it allows us to adjust the boundary conditions to changing experimental conditions. Furthermore, the influence of a spatial correlation in the spontaneous fluctuations in the creation and annihilation rates can be investigated. A comparison with experimental results is presented in ref. [17].

5. The ac impedance

In the calculation given in this section we extend Greebe's impedance calculation of piezoelectric plates [15] by including trough creation and annihilation processes. We extend our earlier calculation [5, 16] by assuming that two types of troughs exist and that the trough velocity may differ from the phase velocity of the acoustic waves.

In this section the effects of space charge, diffusion and the displacement current are taken into account, because the ac impedance can be calculated without neglecting these effects. Thus, the elastic, dielectric and piezoelectric anisotropies are taken into account with respect to the propagation direction and polarization of the acoustic waves. Before calculating the ac impedance, it will be useful to examine more closely the sound-wave-induced electric field strength wave [23–25].

Consider a sound wave which moves along a crystal direction $\boldsymbol{\kappa}$. The sound-wave-induced electric field strength wave is given by

$$\boldsymbol{E}_{\mathrm{ind}}(\boldsymbol{x}, t) = \boldsymbol{E}_{\mathrm{ind}_0}\, \mathrm{e}^{\mathrm{i}(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})}\,, \tag{63}$$

where $\boldsymbol{E}_{\mathrm{ind}_0}$ is an electric field amplitude, independent of $\boldsymbol{x}$ and t, ω is the angular frequency of the sound wave, and

$$\boldsymbol{k} = (k_{\mathrm{RE}} + \mathrm{i}k_{\mathrm{IM}})\boldsymbol{\kappa} . \tag{64}$$

Note that $\boldsymbol{k}$ may be a complex vector, the real part of which ($k_{\mathrm{RE}}\boldsymbol{\kappa}$) is the ordinary wave vector, whereas the imaginary part ($k_{\mathrm{IM}}\boldsymbol{\kappa}$) describes the amplification of the sound wave. Maxwell's equations yield

$$\nabla \times (\nabla \times \boldsymbol{E}_{\mathrm{ind}}) = -\mu_0 \cdot \boldsymbol{\mu}_{\mathrm{r}} \cdot \frac{\partial}{\partial t} \boldsymbol{j}_{\mathrm{ind}} , \tag{65}$$

where μ_0 is the vacuum permeability, $\boldsymbol{\mu}_{\mathrm{r}}$ the relative-permeability tensor and $\boldsymbol{j}_{\mathrm{ind}}$ is the induced total current density. If we assume that $\boldsymbol{\mu}_{\mathrm{r}}$ is the unit tensor, we find the following expression from eqs. (63) and (65):

$$-k^2 \boldsymbol{E}_{\mathrm{ind}} + \boldsymbol{k} \cdot (\boldsymbol{k} \cdot \boldsymbol{E}_{\mathrm{ind}}) = -\mu_0 \,\mathrm{i}\omega \boldsymbol{j}_{\mathrm{ind}} , \tag{66}$$

where

$$k^2 = (\boldsymbol{k} \cdot \boldsymbol{k}) .$$

If we assume that $|k_{\mathrm{Im}}/k_{\mathrm{Re}}| \ll 1$, i.e. the amplification factor is close to unity over one wavelength, and use $\omega^2/k_{\mathrm{RE}}^2 = v_{\mathrm{s}}^2$ with $v_{\mathrm{s}} = |\boldsymbol{v}_{\mathrm{s}}|$ the acoustic phase velocity, eq. (66) can be rewritten as

$$\boldsymbol{E}_{\mathrm{ind}} - \boldsymbol{\kappa}(\boldsymbol{\kappa} \cdot \boldsymbol{E}_{\mathrm{ind}}) = v_{\mathrm{s}}^2 \frac{\mathrm{i}\mu_0}{\omega} \boldsymbol{j}_{\mathrm{ind}} . \tag{67}$$

The left-hand side of eq. (67) is the transverse component of $\boldsymbol{E}_{\mathrm{ind}}$. To estimate the order of magnitude of this component, we approximate $\boldsymbol{j}_{\mathrm{ind}}$ by ($\sigma_0 \boldsymbol{E}_{\mathrm{ind}}$), where σ_0 is a typical conductivity for the samples under study. Eq. (67) now becomes

$$\frac{|\boldsymbol{E}_{\mathrm{ind}} - \boldsymbol{\kappa}(\boldsymbol{\kappa} \cdot \boldsymbol{E}_{\mathrm{ind}})|}{|\boldsymbol{E}_{\mathrm{ind}}|} = v_{\mathrm{s}}^2 \frac{\mu_0 \sigma_0}{\omega} . \tag{68}$$

By inserting reasonable values for the unknowns on the right-hand side of eq. (68), it can be shown that for all practical cases the magnitude of the transverse electric field component can be neglected with respect to $|\boldsymbol{E}_{\mathrm{ind}}|$. (For example, when we use $v_{\mathrm{s}} = 2 \times 10^3\ \mathrm{ms}^{-1}$, $\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H\,m}^{-1}$, $\sigma_0 < 100\ \Omega^{-1}\,\mathrm{m}^{-1}$ and $\omega > 10^5\ \mathrm{s}^{-1}$, the right-hand side becomes smaller than 5×10^{-3}.) Therefore, for the acoustic-wave-induced electric field strength we may write, in good approximation,

$$\boldsymbol{E}_{\mathrm{ind}} = E_{\mathrm{ind}} \boldsymbol{\kappa} . \tag{69}$$

For small deviations Δ from the stationary state we find from eq. (1) the following expression for the electron densities:

$$\Delta n = \Delta n_{\mathrm{d}} + \Delta n_{\mathrm{s}_1} + \Delta n_{\mathrm{s}_2} . \tag{70}$$

By linearizing the master equations (eqs. (9) and (10)) we obtain (disregarding the Langevin source functions, as we are only interested in the ac impedance)

$$\frac{\partial}{\partial t}\Delta n_{t_1} = -\frac{\Delta n_{t_1}}{\tau_1} + \left[\frac{\partial}{\partial E_i}(p_1 - b_1)\right]_{\Delta E_i=0} \Delta E_i - v_{g_i}\frac{\partial}{\partial x_i}\Delta n_{t_1} \tag{71}$$

and

$$\frac{\partial}{\partial t}\Delta n_{t_2} = -\frac{\Delta n_{t_2}}{\tau_2} + \left[\frac{\partial}{\partial E_i}(p_2 - b_2)\right]_{\Delta E_i=0} \Delta E_i + v_{g_i}\frac{\partial}{\partial x_i}\Delta n_{t_2}\,. \tag{72}$$

The linearized equation for deviations in the total current density from the stationary state value becomes (cf. eq. (3))

$$\Delta j_i = -q\bar{n}_{\rm d}\Delta v_{{\rm d}_i} - q\bar{v}_{{\rm d}_i}\Delta n_{\rm d} - qv_{g_i}(\Delta n_{s_1} - \Delta n_{s_2}) + qD_{{\rm n}_{ij}}\frac{\partial}{\partial x_j}\Delta n_{\rm d} + \frac{\partial}{\partial t}\Delta D_i\,. \tag{73}$$

In this equation the ac conductivity of the trapped electrons is neglected (cf. refs. [5, 16]).

Eqs. (2), (4)–(8), (11), (12) and (70)–(73) form a set of 12 linear homogeneous differential equations with 12 variables, $\Delta\boldsymbol{T}$, $\Delta\boldsymbol{S}$, $\Delta\boldsymbol{E}$, $\Delta\boldsymbol{D}$, $\Delta\boldsymbol{u}$, Δj, Δn, $\Delta n_{\rm d}$, Δn_{s_1}, Δn_{s_2}, Δn_{t_1} and Δn_{t_2}, which can be solved by looking for solutions of the type $\exp[i(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})]$. When $k = 0$ we find a trivial solution. When the polarization of the acoustic wave is denoted by the unit vector $\boldsymbol{\pi}$, and when eq. (69) is used the dispersion relation for $k \neq 0$, after some algebra, becomes (note that i when not used as a subscript is the imaginary unity)

$$\rho\omega^2 = c'k^2\,, \tag{74}$$

where

$\boldsymbol{k} = k\boldsymbol{\kappa}\,,\quad c' = c[1 + K_{\rm e}^2\Phi]\,,$

$c = \kappa_i\pi_j c_{ijkl}\pi_k\kappa_l$ the effective elastic constant,

$K_{\rm e} = [e^2/\varepsilon c]^{1/2}$ the effective electromechanical coupling factor,

$e = \kappa_i e_{ijk}\pi_j\kappa_k$ the effective piezoelectric constant (we used $e_{ijk} = e_{ikj}$ in hexagonal crystals),

$\varepsilon = \kappa_i\varepsilon_{ij}\kappa_j$ the effective dielectric constant,

$$\Phi = \left[1 + \left(q\mu\bar{n}_{\rm d} + \frac{\alpha_1(1 + ikD_{\rm n}/(\bar{v}_{{\rm d}_j} - v_{g_j})\kappa_j)}{1/\tau_1 + i(\omega - k_i v_{g_i})} + \frac{\alpha_2(1 + ikD_{\rm n}/(\bar{v}_{{\rm d}_j} + v_{g_j})\kappa_j)}{1/\tau_2 + i(\omega + k_i v_{g_i})}\right)\left(\frac{1}{\varepsilon(k^2D_n + i(\omega - k_i\bar{v}_{{\rm d}_i}))}\right)\right]^{-1}$$

$\mu = \kappa_i\mu_{ij}\kappa_j$ the effective mobility,

$D_{\rm n} = \kappa_i D_{{\rm n}_{ij}}\kappa_j$ the effective diffusion constant,

$$\alpha_1 = qN_1(\bar{v}_{{\rm d}_i} - v_{g_i})\kappa_i\left(\frac{\partial}{\partial E_j}(p_1 - b_1)\right)_{\Delta E_j=0}\kappa_j\,,$$

$$\alpha_2 = -qN_2(\bar{v}_{{\rm d}_i} + v_{g_i})\kappa_i\left(\frac{\partial}{\partial E_j}(p_2 - b_2)\right)_{\Delta E_j=0}\kappa_j\,.$$

From eq. (74) we see that all electro-acoustic effects on wave propagation are described by an effective elastic constant c'. The solution of eq. (74) can now be simplified if it is assumed that the coupling between the acoustic waves and the electrons is small. (For instance, in CdS the maximum value of K_e^2, which is in fact a measure for the ratio of the piezoelectric force and the elastic force, is approximately 0.05.) In other words, the electronic amplification will not change the acoustic wave amplitudes by more than a few percent over one wavelength. In fact this has already been assumed earlier, to arrive at eq. (69). Accordingly, we assume

$$|K_e^2\Phi| \ll 1\,. \tag{75}$$

Since the value of Φ is still unknown, condition (75) should be verified for consistency. Experimental results [5, 16] show that condition (75) is apparently satisfied. This small-coupling approximation implies that c' (and therefore k as well) contains only a relatively small imaginary part. If condition (75) holds, eq. (74) becomes a quadratic equation in k with solutions given by (we used $v_{s_0} = (c/\rho)^{1/2}$, the unstiffened phase velocity [4])

$$k = k' \approx \frac{\omega}{v_{s_0}}(1 - \tfrac{1}{2}K_e^2\Phi') \tag{76}$$

and

$$k = k'' \approx \frac{-\omega}{v_{s_0}}(1 - \tfrac{1}{2}K_e^2\Phi'')\,. \tag{77}$$

Putting $k' \approx \omega/v_s$ in the expression for Φ' and $k'' \approx -\omega/v_s$ in the expression for Φ'' (in fact this means that the imaginary parts of k' and k'' are neglected) and introducing the angular electron diffusion frequency $\omega_D = v_s^2/D_n$, we find

$$\Phi' = \left(\gamma - i\frac{\omega}{\omega_D}\right)\left[\gamma - i\left\{\frac{\omega}{\omega_D} + \frac{1}{\varepsilon\omega}\left(\frac{\alpha_1(1+ia_1)\tau_1}{1+i\omega\theta'\tau_1} + \frac{\alpha_2(1+ia_2)\tau_2}{1+i\omega\theta''\tau_2} + q\mu\bar{n}_d\right)\right\}\right]^{-1}, \tag{78}$$

where

$$a_1 = \frac{\omega}{\omega_D}\frac{v_s}{(\bar{v}_{d_j} - v_{g_j})\kappa_j}\,, \qquad a_2 = \frac{\omega}{\omega_D}\frac{v_s}{(\bar{v}_{d_j} + v_{g_j})\kappa_j}\,;$$

$\theta' = 1 - v_{g_i}\kappa_i/v_s$ and $\theta'' = 1 + v_{g_i}\kappa_i/v_s = 2 - \theta'$ are factors related to the electro-acoustic dispersion (cf. section 7). Furthermore, we used the drift parameter [21]

$$\gamma = 1 - \bar{v}_{d_i}\kappa_i/v_s\,. \tag{79}$$

For Φ'' we obtain

$$\Phi'' = \left(2 - \gamma - i\frac{\omega}{\omega_D}\right)\left[2 - \gamma - i\left\{\frac{\omega}{\omega_D} + \frac{1}{\varepsilon\omega}\left(\frac{\alpha_1(1-ia_1)\tau_1}{1+i\omega\theta''\tau_1} + \frac{\alpha_2(1-ia_2)\tau_2}{1+i\omega\theta'\tau_2} + qu\bar{n}_d\right)\right\}\right]^{-1}, \tag{80}$$

where a_1 and a_2 are defined after eq. (78).

Once we have found the dispersion relations (eqs. (76) and (77)) the solution of our basic equations for any variable $\Delta \boldsymbol{y}(\boldsymbol{x}, t)$, be it a tensor, vector or scalar quantity, at a given angular frequency can be written as a linear combination of plane waves with wave numbers $k = 0$ representing a solution uniform over space, $\boldsymbol{k} = k'\boldsymbol{\kappa}$ representing a plane wave travelling along $\boldsymbol{\kappa}$, and $\boldsymbol{k} = k''\boldsymbol{\kappa}$ representing a plane wave travelling in the opposite direction. So in general we have

$$\Delta \boldsymbol{y}(\boldsymbol{x}, t) = \Delta \boldsymbol{y}_0 \, \mathrm{e}^{\mathrm{i}\omega t} + \Delta \boldsymbol{y}' \, \mathrm{e}^{\mathrm{i}(\omega t - \boldsymbol{k}' \cdot \boldsymbol{x})} + \Delta \boldsymbol{y}'' \, \mathrm{e}^{\mathrm{i}(\omega t - \boldsymbol{k}'' \cdot \boldsymbol{x})} , \tag{81}$$

where $\Delta \boldsymbol{y}_0$, $\Delta \boldsymbol{y}'$ and $\Delta \boldsymbol{y}''$ are independent of $\boldsymbol{x}$ and t.

We now return to our basic equations to derive some useful relations between the plane-wave amplitudes of our variables for each mode separately. For $k = 0$ it follows from eq. (8) that

$$(\Delta S_0)_{ij} = 0 . \tag{82}$$

From eqs. (6), (69) and (82) it follows that

$$\kappa_i (\Delta D_0)_i = \varepsilon \Delta E_0 . \tag{83}$$

Here we used eq. (69):

$$(\Delta E_0)_i = \Delta E_0 \kappa_i . \tag{84}$$

From eqs. (5), (82) and (84) we find

$$\pi_i \kappa_j (\Delta T_0)_{ij} = -e \Delta E_0 . \tag{85}$$

From eqs. (11), (12), (70)–(73), (83) and the definitions in eq. (74) it follows that for $i = 3$,

$$(\Delta j_0)_3 = \left(q \mu_{33} \bar{n}_\mathrm{d} + \frac{\alpha_1 \tau_1}{1 + \mathrm{i}\omega\tau_1} + \frac{\alpha_2 \tau_2}{1 + \mathrm{i}\omega\tau_2} + \mathrm{i}\omega\varepsilon_{33}\right) \Delta E_0 \kappa_3 . \tag{86}$$

Here we also made use of the fact that the off-diagonal elements of the mobility and the dielectric tensor are zero.

For $k = k'$ we find from eqs. (5), (7), (8) and (74) that

$$\pi_i \kappa_j (\Delta T')_{ij} = -\frac{e}{\Phi' K_e^2} \Delta E' , \tag{87}$$

and from eq. (4),

$$(\Delta j')_i = 0 . \tag{88}$$

Analogously, for $k = k''$ we obtain

$$\pi_i \kappa_j (\Delta T'')_{ij} = -\frac{e}{\Phi'' K_\mathrm{e}^2} \Delta E'' , \tag{89}$$

and

$$(\Delta j'')_i = 0\,. \tag{90}$$

The choice of boundary conditions allows us to determine the relative magnitude of the three modes occurring in eq. (81). For two free end-surfaces we take the boundary conditions to be

$$\pi_i \kappa_j [(\Delta T_0)_{ij} + (\Delta T')_{ij}\, e^{-ik'\cdot x} + (\Delta T'')_{ij}\, e^{-ik''\cdot x}]\, e^{i\omega t} \equiv 0; \tag{91}$$

at the cathode, $x_3 = 0$, and at the anode, $x_3 = L$.

So far, the results obtained only describe the behaviour of acoustic waves with a fixed wave-vector direction $\boldsymbol{\kappa}$. If we realize that the results should be averaged over the azimuthal angle ϕ, it follows that the only important components of the electric field strength and the current density are along the x_3 axis.

By integrating the x_3 component of the electric field strength along the direction of the trough velocity, for the alternating voltage with angular frequency ω, developed between the contacts, we find

$$\Delta V = -\int_0^L [\Delta E_0 + \Delta E'\, e^{-ik'\cdot r} + \Delta E''\, e^{-ik''\cdot r}]\, e^{i\omega t}\kappa_3\, dx_3\,, \tag{92}$$

where $\boldsymbol{r}$ is a position vector along the direction of the trough velocity. The magnitude of the phase factor $(\boldsymbol{k}' \cdot \boldsymbol{r})$ at the anode $(x_3 = L)$ is found to be (we use $\boldsymbol{g} = \boldsymbol{r}/r$, a unit vector along $\boldsymbol{r}$)

$$\boldsymbol{k}' \cdot \boldsymbol{r} = k' r \boldsymbol{\kappa} \cdot \boldsymbol{g} = k' L \frac{\boldsymbol{\kappa} \cdot \boldsymbol{g}}{g_3}\,. \tag{93}$$

Thus it is convenient to introduce L_{eff} by defining

$$L_{\text{eff}} = L \frac{\boldsymbol{\kappa} \cdot \boldsymbol{g}}{g_3}\,. \tag{94}$$

From eq. (91) with the help of eqs. (85), (87) and (89) we obtain

$$\Delta E' = K_e^2 \Phi' \left[\frac{e^{-ik'' L_{\text{eff}}} - 1}{e^{-ik' L_{\text{eff}}} - e^{-ik'' L_{\text{eff}}}}\right] \Delta E_0 \tag{95}$$

and

$$\Delta E'' = K_e^2 \Phi'' \left[\frac{e^{-ik' L_{\text{eff}}} - 1}{e^{-ik' L_{\text{eff}}} - e^{-ik'' L_{\text{eff}}}}\right] \Delta E_0\,. \tag{96}$$

With the help of eqs. (92), (95) and (96) ΔV can be expressed in ΔE_0. By integrating the x_3 component of the current density over the cross-sectional area we find, using eqs. (86), (88) and (90), that the total current ΔI is given by

$$\Delta I = A\left(q\mu_{33}\bar{n}_d + \frac{\alpha_1 \tau_1}{1 + i\omega\tau_1} + \frac{\alpha_2 \tau_2}{1 + i\omega\tau_2} + i\omega\varepsilon_{33}\right)\Delta E_0 \kappa_3\, e^{i\omega t}\,. \tag{97}$$

The ac small-signal impedance at angular frequency ω can be calculated with

$$Z(\omega) = -\Delta V/\Delta I. \tag{98}$$

We finally obtain

$$Z(\omega) = L\left[A\left(q\mu_{33}\bar{n}_d + \frac{\alpha_1\tau_1}{1+i\omega\tau_1} + \frac{\alpha_2\tau_2}{1+i\omega\tau_2} + i\omega\varepsilon_{33}\right)\right]^{-1} \times\left\{1 + iK_e^2(k'\Phi'' - k''\Phi')\frac{(e^{-ik'L_{eff}}-1)(e^{-ik''L_{eff}}-1)}{L_{eff}k'k''(e^{-ik'L_{eff}} - e^{-ik''L_{eff}})}\right\}. \tag{99}$$

Substituting reasonable values for the unknowns in eq. (99), we find that the term containing K_e^2 contributes significantly only at frequencies given by

$$f = f_l = (2l+1)v_s/2L_{eff}, \qquad l = 0, 1, 2, 3, \ldots. \tag{100}$$

This is easily seen by putting $k' \approx \omega/v_s$ and $k'' \approx -\omega/v_s$ in eq. (99). It then follows that

$$Z(\omega) \approx L\left[A\left(q\mu_{33}\bar{n}_d + \frac{\alpha_1\tau_1}{1+i\omega\tau_1} + \frac{\alpha_2\tau_2}{1+i\omega\tau_2} + i\omega\varepsilon_{33}\right)\right]^{-1}\left\{1 - K_e^2\frac{v_s}{\omega L_{eff}}(\Phi' + \Phi'')\,\mathrm{tg}\left(\frac{\omega L_{eff}}{2v_s}\right)\right\}. \tag{101}$$

Considering that $v_s/\omega L_{eff} \lesssim 1$ in all practical cases, and that condition (75) holds, it follows that $|Z(\omega)|$ will show narrow maxima at frequencies given by eq. (100). In section 7 it will be shown that L_{eff}/v_s is equal to the trough transit time.

We recall that the ac impedance was calculated with the help of plane waves with wave vectors lying on a cone with half-cone angle δ. In practice, however, it is known that waves with different off-axis angles are involved in the sound amplification process [26]. This distribution of off-axis angles causes the narrow resonances predicted by eq. (100) to be smoothed out somewhat. This smoothing out will occur because a distribution of off-axis angles corresponds to a distribution of trough velocities, and thus to a distribution of trough transit times (cf. refs. [5, 16]).

In section 4 the current noise was calculated. In this calculation the effect of space charge was neglected among other things. As was pointed out in section 4 this neglect removes the piezoelectric coupling of acoustic waves and electric fields. When we put $K_e^2 = 0$ in the expression for the ac impedance (eq. (99) or (101)), $Z(\omega)$ remains unchanged except for frequencies close to the resonance frequencies given by eq. (100): in fact, the transit time resonances disappear. Therefore it is concluded that the absence of transit time resonances in the expression for the current noise (eq. (62)) may be due to the fact that space-charge neutrality was assumed. Apparently, the condition that only frequencies small compared to the dielectric relaxation frequency are considered is not sufficient to allow the assumption of space-charge neutrality. Yet, we assume that the current noise, apart from transit time effects, can be described by eq. (62).

A further discussion about the resonance frequencies given by eq. (100) will be presented in section 7. Apart from these resonances, the behaviour of $Z(\omega)$, as given by eq. (99), is of interest in three limiting cases for our ac-impedance measurements:

(i) $Z(\omega) \to L/A(q\mu_{33}\bar{n}_d + \alpha_1\tau_1 + \alpha_2\tau_2) \quad \text{for } \omega \to 0;$ (102)

$$\text{(ii)}\quad Z(\omega)\to L/\mathrm{i}\omega A\varepsilon_{33}\qquad \text{for } \omega \gg \left|\left(q\mu_{33}\bar{n}_{\rm d}+\frac{\alpha_1\tau_1}{1+\mathrm{i}\omega\tau_1}+\frac{\alpha_2\tau_2}{1+\mathrm{i}\omega\tau_2}\right)\left(\frac{1}{\varepsilon_{33}}\right)\right|\,, \tag{103}$$

which corresponds to the ac impedance of a device with capacity $C=\varepsilon_{33}A/L$;

$$\text{(iii)}\quad Z(\omega)\approx L/Aq\mu_{33}\bar{n}_{\rm d} \tag{104}$$

at intermediate frequencies, where

$$\left|\frac{\alpha_1\tau_1}{1+\mathrm{i}\omega\tau_1}+\frac{\alpha_2\tau_2}{1+\mathrm{i}\omega\tau_2}+\mathrm{i}\omega\varepsilon_{33}\right| \ll q\mu_{33}\bar{n}_{\rm d}\,.$$

Furthermore, at low frequencies we generally have two roll-offs, determined by the relaxation times τ_1 and τ_2; at intermediate frequencies we expect a plateau, which is determined by free carriers only (this is a consequence of the assumption that the ac conductivity of trapped electrons can be neglected); at high frequencies we find the familiar dielectric roll-off. Measurements of the ac impedance of CdS single crystals will be discussed in ref. [18].

6. The attenuation coefficients

From the calculation of the ac impedance presented in section 5 one can obtain some additional interesting results. The imaginary parts of the wave numbers k' and k'' (cf. eqs. (76) and (77)) yield the attenuation coefficients $\alpha_{\rm e}'$ of waves travelling along $\boldsymbol{\kappa}$, and $\alpha_{\rm e}''$ of waves travelling in the opposite direction, respectively. (Note that amplification occurs when the attenuation coefficient is negative.) It turns out that

$$\begin{aligned}\alpha_{\rm e}'(\omega) &= -\mathrm{Im}(k') = \tfrac{1}{2}K_{\rm e}^2/\varepsilon v_{\rm s_0}\\
&\quad\times\left[\gamma\left\{q\mu\bar{n}_{\rm d}+\frac{\alpha_1\tau_1(1+a_1\omega\tau_1\theta')}{1+\omega^2\tau_1^2\theta'^2}+\frac{\alpha_2\tau_2(1+a_2\omega\tau_2\theta'')}{1+\omega^2\tau_2^2\theta''^2}\right\}\right.\\
&\quad\left.-\frac{\omega}{\omega_{\rm D}}\left\{\frac{\alpha_1\tau_1(a_1-\omega\tau_1\theta')}{1+\omega^2\tau_1^2\theta'^2}+\frac{\alpha_2\tau_2(a_2-\omega\tau_2\theta'')}{1+\omega^2\tau_2^2\theta''^2}\right\}\right]\\
&\quad\times\left[\left\{\gamma+\frac{1}{\varepsilon\omega}\left[\frac{\alpha_1\tau_1(a_1-\omega\tau_1\theta')}{1+\omega^2\tau_1^2\theta'^2}+\frac{\alpha_2\tau_2(a_2-\omega\tau_2\theta'')}{1+\omega^2\tau_2^2\theta''^2}\right]\right\}^2\right.\\
&\quad\left.+\left\{\frac{\omega}{\omega_{\rm D}}+\frac{1}{\varepsilon\omega}\left[q\mu\bar{n}_{\rm d}+\frac{\alpha_1\tau_1(1+a_1\omega\tau_1\theta')}{1+\omega^2\tau_1^2\theta'^2}+\frac{\alpha_2\tau_2(1+a_2\omega\tau_2\theta'')}{1+\omega^2\tau_2^2\theta''^2}\right]\right\}^2\right]^{-1},\end{aligned} \tag{105}$$

where

$$a_1=\frac{\omega}{\omega_{\rm D}}\frac{v_{\rm s}}{(\bar{v}_{{\rm d}_i}-v_{{\rm g}_i})\kappa_i} \tag{106}$$

and

$$a_2=\frac{\omega}{\omega_{\rm D}}\frac{v_{\rm s}}{(\bar{v}_{{\rm d}_i}+v_{{\rm g}_i})\kappa_i}\,. \tag{107}$$

Analogously, we find

$$\alpha_e'' = \mathrm{Im}(k'') = \tfrac{1}{2}K_e^2/\varepsilon v_{s_0}$$
$$\times\Big[(2-\gamma)\Big\{q\mu\bar{n}_d + \frac{\alpha_1\tau_1(1-a_1\omega\tau_1\theta'')}{1+\omega^2\tau_1^2\theta''^2} + \frac{\alpha_2\tau_2(1-a_2\omega\tau_2\theta')}{1+\omega^2\tau_2^2\theta'^2}\Big\}$$
$$+\frac{\omega}{\omega_D}\Big\{\frac{\alpha_1\tau_1(a_1+\omega\tau_1\theta'')}{1+\omega^2\tau_1^2\theta''^2} + \frac{\alpha_2\tau_2(a_2+\omega\tau_2\theta')}{1+\omega^2\tau_2^2\theta'^2}\Big\}\Big]$$
$$\times\Big[\Big\{2-\gamma-\frac{1}{\varepsilon\omega}\Big[\frac{\alpha_1\tau_1(a_1+\omega\tau_1\theta'')}{1+\omega^2\tau_1^2\theta''^2} + \frac{\alpha_2\tau_2(a_2+\omega\tau_2\theta')}{1+\omega^2\tau_2^2\theta'^2}\Big]\Big\}$$
$$+\Big\{\frac{\omega}{\omega_D}+\frac{1}{\varepsilon\omega}\Big[q\mu\bar{n}_d + \frac{\alpha_1\tau_1(1-a_1\omega\tau_1\theta'')}{1+\omega^2\tau_1^2\theta''^2} + \frac{\alpha_2\tau_2(1-a_2\omega\tau_2\theta')}{1+\omega^2\tau_2^2\theta'^2}\Big]\Big\}^2\Big]^{-1}. \tag{108}$$

Due to the complexity of expressions (105) and (108) no simple predictions about the voltage and frequency dependence of the attenuation coefficients can be given without detailed knowledge of the various, generally voltage-dependent parameters. We confine ourselves to some general remarks.

The magnitude of the attenuation coefficients appears to be very sensitive to the magnitude of the dispersion factors θ' and θ'' (cf. eq. (78)). In the following section it will be shown that both θ' and θ'' are essentially frequency dependent as a consequence of electro-acoustic frequency dispersion.

It should be noted that these expressions reduce to White's much simpler results for the linear attenuation coefficients [1], if we put $\alpha_1\tau_1 = 0$, $\alpha_2\tau_2 = 0$ and replace $(q\mu\bar{n}_d)$ by the conductivity $(q\mu\bar{n})$.

7. Dispersion effects

In this section we present the calculation of the group velocity $\boldsymbol{v}_g$ of an acoustic wave packet, resulting from a collection of plane acoustic waves with wave vectors close to $\boldsymbol{k} = k\boldsymbol{\kappa}$ (k is real). The group velocity of this wave packet is then found with

$$\boldsymbol{v}_g = \nabla_k \omega(\boldsymbol{k}) . \tag{109}$$

When we use

$$\omega(\boldsymbol{k}) = \boldsymbol{k}\cdot\boldsymbol{v}_s = kv_s , \tag{110}$$

eq. (109) becomes

$$\boldsymbol{v}_g = \nabla_k (kv_s) . \tag{111}$$

Because of rotational symmetry around the c axis, using the off-axis angle δ, we can rewrite eq. (111) as

$$\boldsymbol{v}_g = \Big(v_s + k\frac{\partial v_s}{\partial k}\Big)\boldsymbol{\kappa} + \frac{\partial v_s}{\partial \delta}\boldsymbol{\kappa}_\delta , \tag{112}$$

where $\boldsymbol{\kappa}_\delta$ is a unit vector perpendicular to $\boldsymbol{k}$, lying in the plane of $\boldsymbol{k}$ and the x_3 axis, defined by

$$\boldsymbol{\kappa}_\delta = \frac{\boldsymbol{\kappa}\times(\boldsymbol{\kappa}\times\boldsymbol{x}_{3_0})}{|\boldsymbol{\kappa}\times(\boldsymbol{\kappa}\times\boldsymbol{x}_{3_0})|} , \tag{113}$$

where $\boldsymbol{x}_{3_0}$ is a unit vector along the x_3 axis (c axis).

In general the phase velocity $\boldsymbol{v}_s$ will be a function of k and δ. In fact, because v_s is determined by the real part of the elastic constant, with the help of the definitions in eq. (74) we find

$$v_s = v_{s_0}[1 + \tfrac{1}{2}K_e^2\,\mathrm{Re}(\Phi)] . \tag{114}$$

We recall that v_{s_0} is the unstiffened phase velocity given by $v_{s_0} = (\kappa_i \pi_j c_{ijkl} \pi_k \kappa_l / \rho)^{1/2}$. By substituting eq. (114) into eq. (112) we obtain:

$$\boldsymbol{v}_g = v_s\left\{1 + \frac{k}{v_{s_0}}\frac{\partial}{\partial k}\, v_{s_0} + \frac{1}{2}\,k\,\frac{v_{s_0}}{v_s}\,K_e^2\,\frac{\partial}{\partial k}\,(\mathrm{Re}(\Phi))\right\}\boldsymbol{\kappa} + v_s\left\{\frac{1}{v_{s_0}}\frac{\partial}{\partial \delta}\, v_{s_0} + \frac{1}{2}\frac{v_{s_0}}{v_s}\frac{\partial}{\partial \delta}\,(K_e^2\,\mathrm{Re}(\Phi))\right\}\boldsymbol{\kappa}_\delta\,. \tag{115}$$

The various terms in this equation can be identified as follows:

$\frac{k}{v_{s_0}}\frac{\partial}{\partial k}\, v_{s_0}\boldsymbol{\kappa}$	elastic frequency dispersion ,
$\frac{1}{2}\,k\,\frac{v_{s_0}}{v_s}\,K_e^2\,\frac{\partial}{\partial k}\,(\mathrm{Re}(\Phi))\boldsymbol{\kappa}$	electro-acoustic frequency dispersion ,
$\frac{1}{v_{s_0}}\frac{\partial}{\partial \delta}\, v_{s_0}\boldsymbol{\kappa}_\delta$	elastic angular dispersion ,
$\frac{1}{2}\frac{v_{s_0}}{v_s}\frac{\partial}{\partial \delta}\,(K_e^2\,\mathrm{Re}(\Phi))\boldsymbol{\kappa}_\delta$	electro-acoustic angular dispersion .

The elastic frequency dispersion can be neglected, since in the continuum approximation (usually valid for elastic waves with frequencies below 10^2 or 10^3 GHz) v_{s_0} is independent of k [27].

The elastic angular dispersion can be calculated if the elastic constants are known. For instance, for CdS [26] one finds that the elastic angular dispersion for transverse waves (T_2-mode) has a maximum at $\delta = 20°$. In that case the angle between the direction of the group velocity and the c axis is 38°. For longitudinal waves the elastic angular dispersion is somewhat less pronounced.

It is much more complicated to calculate the electro-acoustic dispersion effects. The reason is that the function Φ defined in eq. (74), which is needed for the calculation of $\boldsymbol{v}_g$, contains $\boldsymbol{v}_g$. Therefore, it will be necessary to use numerical methods to find $\boldsymbol{v}_g$. Keller [26] estimated, under some limiting conditions, the magnitude of the electro-acoustic dispersion using the linear theory of White [1]. It was shown that electro-acoustic angular dispersion depends strongly on the acoustic frequency. The electro-acoustic frequency dispersion generally depends on the acoustic frequency, and on the direction and is of the order of K_e^2.

Important quantities appearing in the attenuation coefficients (cf. section 6) are the dispersion factors θ' and θ'', defined in eq. (78). From eqs. (78) and (115) it is found that

$$\theta' = -\frac{1}{2}K_e^2 k\,\frac{\partial}{\partial k}\,(\mathrm{Re}(\Phi))\,, \qquad (\theta'' = 2 - \theta')\,. \tag{116}$$

To find θ' (and θ''), eq. (116) should be solved (note that the function Φ contains θ' and θ''; cf. eq. (78)). Additional complications in the evaluation of θ' and θ'' are the frequency and directional dependence of these quantities, as can be seen from eq. (116).

When we use Keller's estimate [26] of the electro-acoustic frequency dispersion, it is found that

$$|\theta'| \leqq K_e^2/4\,; \qquad \theta'' \approx 2\,. \tag{117}$$

It can be shown that the magnitude of the attenuation coefficients (cf. section 6) is strongly dependent on $|\theta'|$, even if $|\theta'| \ll K_e^2/4$.

Finally, we discuss the resonance frequencies predicted in the ac impedance, which are given by eq. (100). If we use eqs. (94), (100) and (115), and neglect the elastic and electro-acoustic frequency dispersion in the phase velocity, for the resonance frequencies we find

$$f_l = (2l+1)\frac{v_{g3}}{2L} = \frac{(2l+1)}{2}\,\tau_t^{-1}\,; \qquad l = 0, 1, 2, 3, \ldots, \tag{118}$$

with τ_t the trough transit time as defined in eq. (61).

As will be discussed in ref. [18], the effect of electro-acoustic dispersion may influence the magnitude of the trough transit velocity v_{g3}.

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